Applications of Sequences and Series

1. A runner begins training by running 5 mi. one week. The second week she runs a total of 6.5 mi. The third week she runs 8 mi. Assume this pattern continues.
   - How far will she run in the tenth week?
   - At the end of the tenth week, what will be the total distance she has run since she started training?
   - Express the total distance with summation notation ($\sum$).

2. A superball is dropped from a height of 2 m and bounces 90% of its original height on each bounce.
   - When it hits the ground for the eighth time, how far has it traveled?
   - How high off the floor is the ball at the top of the eighth bounce?

3. A snail is crawling straight up a wall. The first hour it climbs 16 inches, the second hour it climbs 12 inches, and each succeeding hour, it climbs only three-fourths the distance it climbed the previous hour. Assume the pattern continues.
   - How far does the snail climb during the seventh hour?
   - What is the total distance the snail has climbed in seven hours?
   - Express the total distance with summation notation ($\sum$).

4. Suppose on Jan. 1 you deposit $1.00 in an empty piggy bank. On Jan. 8 you deposit $1.50; on Jan. 15 you deposit $2.00; and each week thereafter you deposit $0.50 more than the previous week.
   - What kind of sequence do these deposits generate?
   - What amount will you deposit in the 52^{nd} week?
   - What is the total in the piggy bank at the end of these 52 weeks?

5. Carla's Clothing Shop opened eight years ago. The first year she made $3,000 profit. Each year thereafter her profits averaged 50% greater than the previous year.
   - How much profit did Carla earn during her 18^{th} year of business?
   - What was the total amount of profit Carla earned over her first 18 years?

6. A ball on a pendulum moves 50 cm on its first swing. Each succeeding swing it moves 0.9 the distance of the previous swing.
   - Write the first six terms of the sequence generated.
   - Assuming the pattern continues, how far will the ball travel before coming to rest?

7. Find the sum of the odd integers from 25 to 75, inclusive.

Applications of Sequences and Series - Solutions

1.) a. $a_{10} = 5 + 1.5(10-1) = \boxed{18.5 \text{ mi}}$

b. $S_{10} = \frac{10(5+18.5)}{2} = \boxed{117.5 \text{ mi}}$

c. $\sum_{K=1}^{10} [5 + 1.5(K-1)]$

2.) a. $S_8 = 2\left[\frac{1-0.9^8}{1-0.9}\right] = 11.39 \text{ m}$

b. $S_9 = 2\left[\frac{1-0.9^9}{1-0.9}\right] = 12.25 \text{ m}$

3.) a. $a_7 = 16\left(\frac{3}{4}\right)^{7-1} = 2.85 \text{ in}$

b. $S_7 = 16\left[\frac{1-\frac{3}{4}^7}{1-\frac{3}{4}}\right] = 55.46 \text{ in}$

c. $\sum_{K=1}^{7} 16\left(\frac{3}{4}\right)^{n-1}$

7.) $\sum_{K=1}^{26} [25 + 2(K-1)]$

$S_{26} = \frac{26(25+75)}{2}$

$S_{26} = 1300$

4.) a.) Arithmetic

b.) $a_{52} = 1 + 0.5(52-1) = \$26.50$

c.) $S_{52} = \frac{52(1+26.50)}{2} = \$715$

5.) a.) $a_{18} = 3{,}000(1.5)^{18-1} = \$2{,}955{,}780$

b.) $S_{18} = 3000\left[\frac{1-(1.5)^{18}}{1-1.5}\right] = \$8{,}861{,}350$

6.) a.) $a_1 = 50$, $a_2 = 50(.9) = 45$, $a_3 = 40.5$, $a_4 = 36.45$

$a_5 = 32.805$, $a_6 = 29.5245$

b.) $\frac{a}{1-r} = \frac{50}{1-0.9} = 500 \text{ cm}$